Exploring Hyperobjects: A Metaphor of Higher Dimensions

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Abstract

The idea that 3-D projections of 4-D objects are analogous to 2-D projections of 3-D objects is an old one, but the ability to represent and explore these 3-D images in detail was impossible before the advent of computers and 3-D modeling programs. This paper will define "hyperobjects" as a loose 3-D analogy of the 2-D images. It will explain how these hyperobjects must be viewed from the inside and how reflective materials provide a simulation of fourth dimensionality. It will describe how images produced by various classes of hyperobjects differ and how they can generate some very sophisticated images that can certainly be taken as representing recursive mathematical concepts as well as higher spatial dimensions .

1. Introduction

1.1 The Analogy. "The concept of the fourth (spatial) dimension is usually surrounded by mystery and suspicion. How dare we, creatures of length, height, and width, speak of four-dimensional space? Is it possible by using all our three-dimensional intelligence to imagine a super-space of four dimensions?…We *do*, in a certain sense, squeeze three-dimensional bodies into a plane by drawing a picture of them." [1] A two-dimensional drawing of a cube is a square within a square (Figure 1). By analogy a three-dimensional "drawing" of a fourth-dimensional cube would be a cube within a cube. The fact that I can illustrate this in two dimensions shows the power of human imagination (Figure 2). These fourth-dimensional objects represented in three dimensions can be called *hyperobjects*.

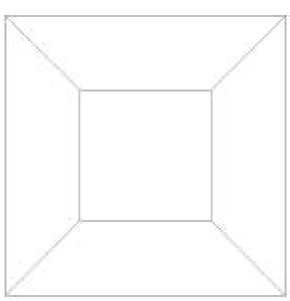

Figure 1: *A Cube in 2-D*

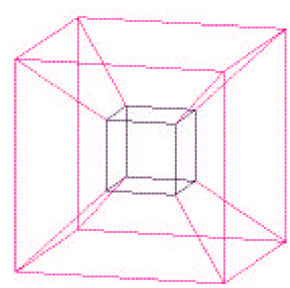

Figure 2: *A (2-D representation of) a Super-cube in 3-D*

1.2 Stretching the Analogy. Does the same analogy work for other geometric objects? Consider a sphere. At first glance it would seem that a sphere within a sphere would constitute a hypersphere. But on returning to the original 2-D view, we find that a sphere is not represented as a circle within a circle. A circle within a circle is the representation of a cylinder. But only as seen by looking at it from the end. If it is viewed from the side the circles become ellipses connected by lines. Depending on the height of the cylinder and the angle of view they may or may not be intersecting. The 2-D representation presents both sides of a sphere as a single circle, but the perspective distortion would be different for each side. In the case of a sphere the angle of view does not change the representation. Viewed from other that straight on the representation of a cube can take many other forms. A single three dimensional object must be represented by a whole class of 2-D drawings. This is further complicated when we look at objects that are not regular. Non-cubic rectangular solids can vary in length, width, and height, which in turn affect the shape. A cylinders and cones change as their diameters and heights change. Therefore, it must be true that a single fourth dimensional object must be represented by a whole class of 3-D objects.

For this reason it is expedient to name hyperobjects after the objects in the representation rather than the higher dimensional object being represented. By this convention a sphere within a sphere would be a hypersphere rather than a hypercylinder; a cylinder within a cylinder would be a hypercylinder, etc.

1.3 Stretching the Analogy Further. If an analogy is to be made between 2-D and 3-D representations, then viewer perception must be analyzed. How does a Flatlander perceive the 2-D cube? Presuming that the resident of Flatland has depth perception and mobility, but not X-ray vision, upon examining the cube he will find it no different than a square (Figure 3).That is of course if his point of view is outside the box.

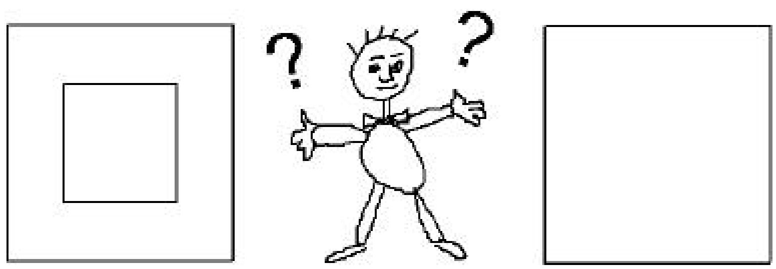

Figure 3: *A Flatlander's point of view.*

In order for him to see the difference his point of view must be inside the box. (Thinking outside the box is not necessarily more creative.) Notice that the lines connecting the corners of the squares are gone. These lines are significant in that they represent edges of the cube. But, what can the Flatlander see. If we examine our perception of lower dimensions we realize that we cannot perceive two dimensional objects; A square must have some thickness to exist in a 3-D world. By extension neither we nor the Flatlander can see a line with only one dimension. In this stretched analogy I have eliminated the surfaces that connect the 3-D shapes that make up the hyperobjects. (In reality these are not simply surfaces, but sides of additional 3-D projections of parts of the 4-D object, just as the lines removed in the Flatland version of the cube are not simply lines but sides of trapezoidal projections of additional sides of the cube. In order for these to be included there have to be two or more objects in the same space. Since this is clearly impossible, it is more convenient to eliminate them.)

2. Representing Higher Dimensions

2.1 Virtual Hyperobjects. Until recently representing hyperobjects in 3-D has been problematic. In the case of the cube, for example, It could only be drawn in 2-D as in Figure 2, or “drawn” in 3-D as (literally) a wire-frame model. With the advent of computers and powerful 3-D modeling and rendering tools it has become possible to create virtual hyperobjects. Although practically these can only be presented on a 2-D screen, walk-through and examine tools allow them to behave very much as true 3-D objects. Furthermore, view points can be created without the intrusion of a viewer, and these can easily be made to be inside the object itself. (Figure 4)

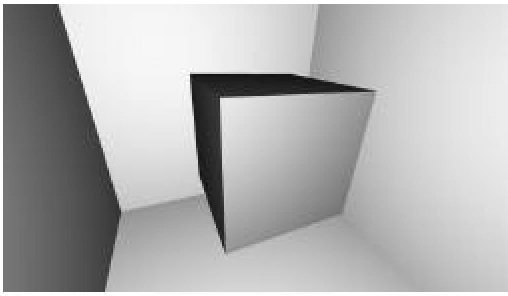

Figure 4: *A 3-D hypercube*

Figure 5: *A mirror hypercube*

2.2 The Ontology of Virtual Objects. The simple representation of a cube within a cube is a useful metaphor for a hyperobject, but it isn't yet very ontologically satisfying. How can it be made so? Two cubes could be Boolean added together to make a single object. This works if the cubes are surfaces or shells, if, however, they are solids it simply gives a cube identical to the larger cube. Upon looking at the nature of 3-D computer modeling some very interesting things become clear. Objects using an ACIS modeling engine behave as if they were solid objects; sliced in two they yield solid objects not surfaces or shells. In every way that is, but one. They can be viewed from the inside. If we put one ACIS solid inside another, the smaller can be subtracted from the larger to give a single object with a void in it. This object can be viewed from inside the resulting solid part. This, I would suggest, is an ontologically satisfying hyperobject. (And it can lead to the metaphysical question of whether all mater is simply holes in the ether.)

2.3 Adding Another Dimension. I've made an analogy, stretched it, stretched it further, and insisted on ontological purity. The resulting virtual hyperobject may be pure, but how can it be made interesting and how can it represent a fourth dimension? The element of time is already necessary for the examination of the hyperobject, it has to turn or be seen from a changing point of view to express its true 3-D nature on a 2-D surface. So, using artistic license, I'll try representing its fourth dimensionality in the metaphor of a mirror. This can be done in a 3-D computer model simply by assigning a mirror material. An additional virtue of the computer model is that there is no intrusion of the viewer's reflection. As Figure 5 shows this introduces a whole new dimension. (Although, in the figurative and metaphorical sense this is certainly true, I won't try to defend it in the logic of the 2-3-4-D analogy. In the artistic sense it doesn't violate the ontological purity of the virtual hyperobject.) I didn't create this concept of the virtual hyperobject and then make the objects. Rather, I created the objects, found them extremely interesting, and then refined the concept as post factum justification.

3. Modeling Hyperobjects

3.1 Simple Hyperobjects. I started with what I call "simple hyperobjects." These are objects in which the subtracted interior is simply a scaled version of the outer object. The cube above is a simple hyperobject. Initially I tried to keep everything as simple as possible. One of the first things I discovered was that purely ambient light does not produce reflections in a pure mirror material. I deliberately kept the material simple: plain mirror, black mirror or gold mirror. I tried to use minimum lighting: one or two point lights, or a single headlight. I could introduce surprising colors simply with a colored light. The hypercube produced a myriad of cube images in its reflections, but lighting it without extreme glare proved very difficult. This seems to be because of the right angles of the faces. The hypersphere was relatively uninteresting; I think this is due to a combination of its lack of distinct facets and its total symmetry The hpersphere is always rotated the same in relation to the viewer. Only the orientation of the light source affects the rendering. (Figures 6 and 7)

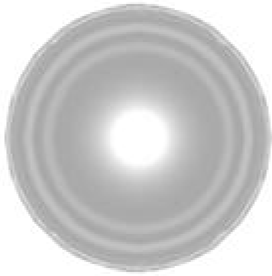

Figure 6: *Hypersphere with a headlight*

Figure 7: *Hypersphere with a point light*

3.2 Faceted Hyperobjects. All hyperpolyhedra render as crystallographic images. What begins to become clear in these renderings is the recursive structure of the reflections. The renderings reveal self-similarity at different degrees of magnification. More complicated hyperpolyhedra tend to produce more complicated images, not necessarily more interesting ones, although they definitely tend to complicate the rendering process itself. (Figures 8 and 9)

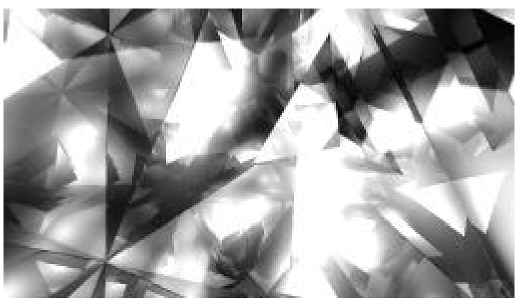

Figure 8: *Hypertetrahedron*

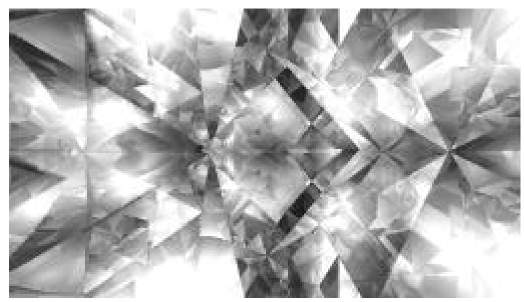

Figure 9: *Hyperoctahedreon*

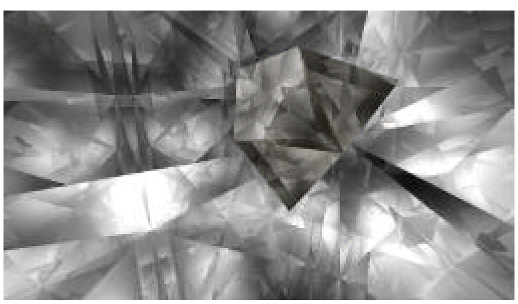

Figure 10: *Hyper-stellated-tetrahedron. View 1.*

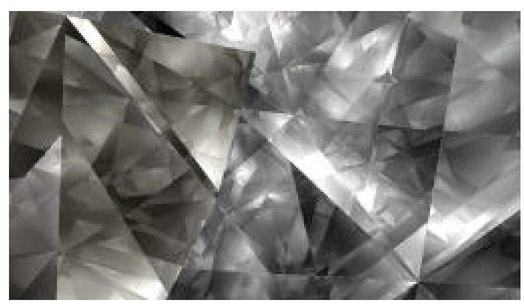

Figure 11: *Hyper-stellated-tetrahedron. View 2.*

As the shapes become more faceted and less symmetrical, point of view begins to make more of a difference in the renderings. A single model can produce a multitude of remarkable images. Because of this, animating rotations of the model or camera and saving the individual images is a good way to explore them. (Figures 10 and 11) Since the facets of a regular polyhedron are all identical regular polygons, self similarity tends to be constant across reflective levels. Zooming in or out doesn't change the appearance much. Neither do the positions of the lights. Colored lights produce quite beautiful renderings. The resemblance of some of these images+ to the analytical cubism of Picasso and Brach is noteworthy.

3.3 Curved Surfaces. Because of their total symmetry, spheres are not particular interesting hyperobjects. But other solids with curved surfaces do produce some very interesting reflections. These change greatly according to point of view. In rendering faceted hyperobjects, the recursive images reflect the shape of the object itself. Introducing curved surfaces changes this. There is self-similarity but the shapes seem to be products of rendering the reflections. The 3-D modeling process uses approximations of curves to some degree and these show up to some extent in the renderings. Although pseudofacets tend to distort the images somewhat, they are not the primary cause of the recursiveness, which is, of course, the result of reflections of reflections ad infinitum. (Figures 13, 14, 15, and 16)

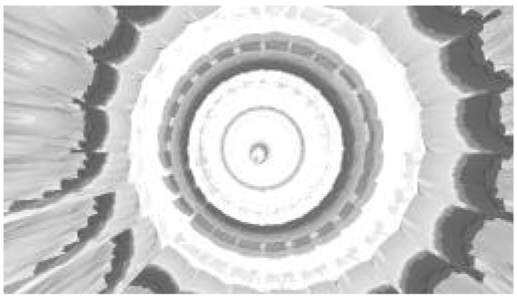

Figure 12: *Hypercone. View 1.*

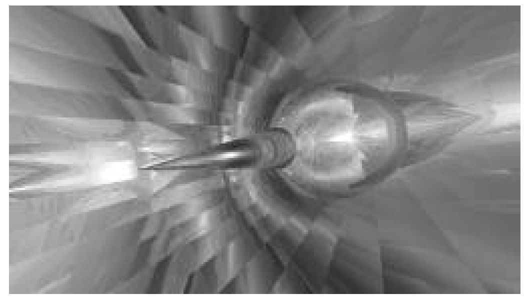

Figure 13: *Hypercone.View2.*

Figure 14: *Hypercylinder. View 1.*

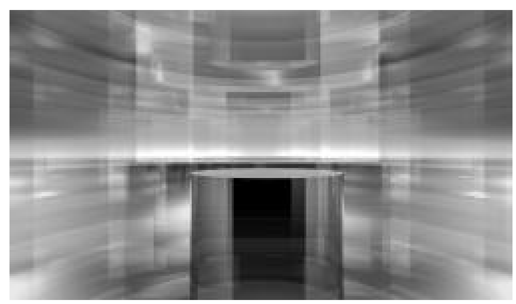

Figure 15: *Hypercylinder. View2*

3.4 Hyperellipsoids. The cone and the cylinder have flat facets, but these are not the only source of interest in the hyper reflections. Although the hypersphere proved relatively uninteresting, some of the most interesting images of simple hyperobjects come from hyperellipsoids. Borrowing from UFOlogy, I'll call the two basic shapes a cigar ellipsoid (Figures 16 and 17.) and a saucer ellipsoid (Figures 18 and 19.) They were created by revolving an ellipse around its large and small diameters respectively. Because of non-uniform distortions the reflected shapes become very strange as the depths increase. Because the images change almost violently depending on point of view, using animations to produce multiple images, then extracting the most interesting is a practical way to proceed. As art these images are, therefore, almost found objects. They are to be discovered within the hyperobject rather than being created from it.

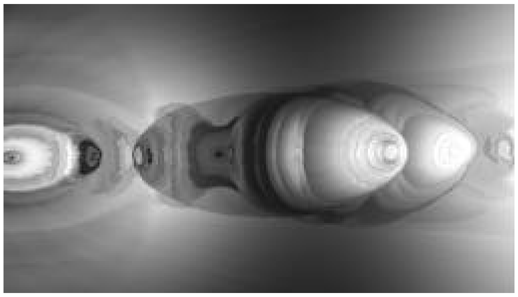

Figure 16: *Hyperellipsoid. Cigar. View 1.*

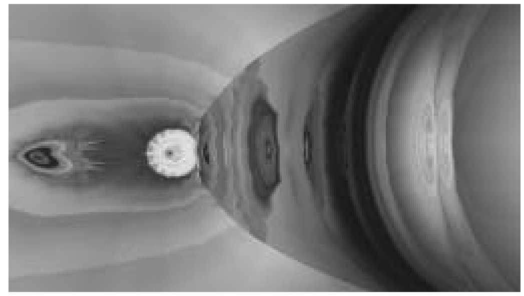

Figure 17: *Hyperellipsoid. Cigar. View 2.*

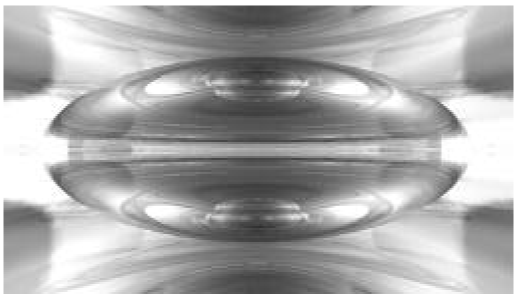

Figure 18: *Hyperellipsoid. Saucer. View 1.*

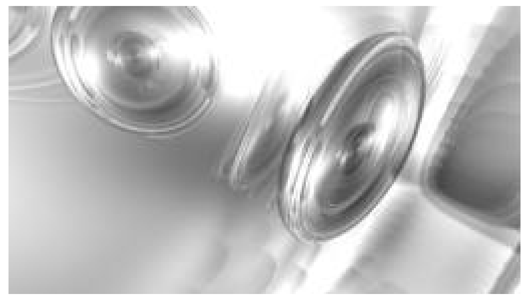

Figure 19: *Hyperellipsoid. Saucer. View 2.*

4. Hybrid Hyperobjects

4.1 Rationalization. Stretching an analogy created justification for simple hyperobjects. But if we consider hybrid hyperobjects, a cube within a sphere for example, how can it be justified? Consider the prism created by joining a circle to a square. (Figure 20.) Viewed in perspective from one end it is a square within a circle. (Figure 21.) Viewed from the other end it would be a circle within a square. Viewed from slightly different angle it would give intersecting shapes. This is ample rationalization for hybrid hyperobjects, of the object within-an-object type and the intersecting objects type (Figure 23.) I will call hyperobjects of both these types, hyperprisims.

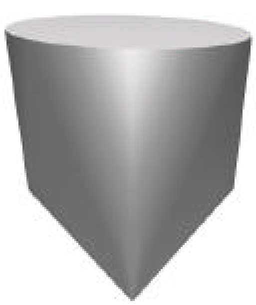

Figure 20: *A circle-square prism.*

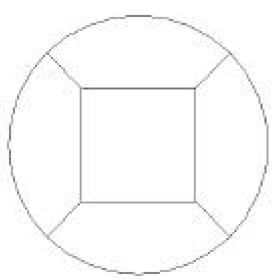

Figure 21: *This prism in. one point perspective*

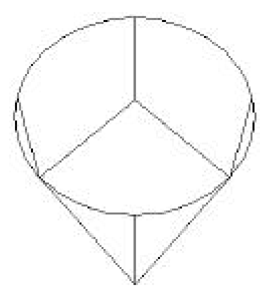

Figure 22: *Prism from a slightly different angle*

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4.2 Hyperprisims. It is not my intention here to develop a descriptive taxonomy of hyperobjects. Simple hyperobjects can be named for their 3-D objects. Even so this ignores the relative sizes of the inner and outer objects and the proportions of irregular objects. Expanding on the above definition of a hyperprism, I will say that it is a class of hyperobjects created from two different intersecting 3-D objects. This is a very broad class, but since the 3-D drawings of the hyperobjects are saved and can be measured, they will serve as their own descriptions, although they still lack any logical filing system. Within a rigorously systematic approach points of view and light positions must also be saved and documented. The sphere-cube hyperprism of Figure 23 is analogous to the projection in Figure 22.

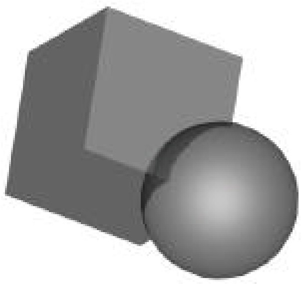

Figure 23: *A sphere-cube hyperprism*

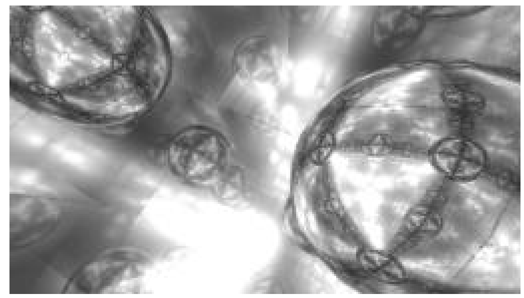

Figure 24: *View from within the cube*

Rendered from within the sphere, this hyperprism is of little interest; viewed from inside the intersection of the sphere and cube, this it is somewhat more interesting. However, when the view point is placed inside the cube a very complex recursive structure becomes apparent. Worlds recede ad infinitum and upon close examination each of these worlds reveals another set of similar receding worlds. That could be a description of the Mandelbrot Set!

4.3 Complex Hyperobjects. After modeling a simple hyperellipsoid I modeled an ellipsoid containing two smaller ellipsoids. This seemed logical in that an ellipse has two focal points, but other than that I did not try to justify it through analogy. I dubbed it a complex hyperellipsoid. This object proved to be so interesting that I developed a complicated camera path to explore it through animation, saving all the individual frames. I placed the entire object in a spherical environment in order to illustrate the camera entering the ellipsoid, and because the camera passes through the smaller ellipsoids I put a sphere in one and a torus in the other. I did this without feeling any further need for justification other than visual interest. I call this project, *Upon Shooting the Moon, I Discovered a Strange Dimension.* In the exterior viewing environment the ellipsoid looks like a moon. The images rendered by viewing from within were so entirely unpredictable that I feel like they were there to be discovered. Figures 25, 26, 27 and 28 are four frames picked more or less at random from the video. None of them is a view from the interior of the small ellipsoids.

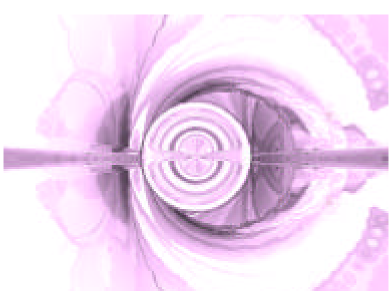

Figure 25: *Complex Hyperellipsoid*

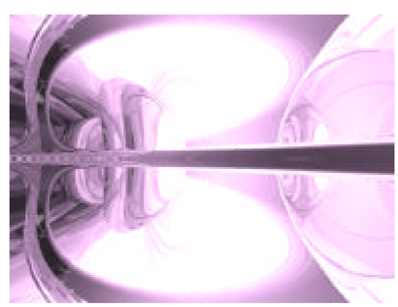

Figure 26: *Complex Hyperellipsoid*

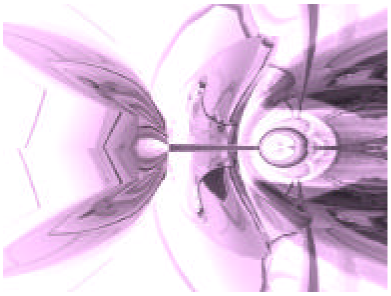

Figure 27: *Complex Hyperellipsoid*

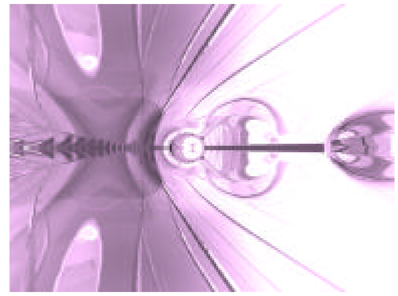

Figure 28: *Complex Hyperellipsoid*

In this remarkable set of images, generated only from an ellipsoid, a strange world becomes visible. Here again, there are similarities to the Mandlebrot Set, in the self-similarity over a range of scales and in some of the shapes themselves.

5. Conclusions

5.1 Metaphysical Speculations. The similarities of some of the images of these virtual hyperobjects to the Mandelbrot Set and other recursive self-similar structures is remarkable. Is it possible that these are images of hyperobjects? Suppose for a moment that there is some view of some 3-D hyperobject that yields a particular 2-D set. Can that object be determined from that set? What is the nature of the 4-D object represented by this hyperobject? Are these sets windows into higher dimensions? These questions may be unanswerable; they certainly are unanswerable by me.

5.2 Further Explorations. I have only just begun exploring hyperobjects. What began on a whim has become a major project. Since I have a virtually unlimited source of images, I think it is conceptually important to document the objects themselves and preserve that documentation along with the images.

References

[1] George Gamow, *One, Two, Three…Infinity*, The Viking Press, New York, 1962